Forum on Science & Technology

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Proof and Truth in Mathematics

The idea of "proof" is the guiding light of mathematics. No matter how many examples you can give for the reality of your theorem, if you cannot offer a valid proof, then your theorem is merely a conjecture. After confidently mastering elementary algebra, most students are taken aback by the difference between algebra and geometry. Elementary algebra is taught as a tautology. Expressions are set equal to each other, and these equations are manipulated to obtain the desired reduction in complexity, such as finding the value of a variable (equating the variable to a number). Geometry, however, is an exercise in logic. In a geometric proof, each step follows logically from the others, and there is a chain of truth that extends from beginning to end.

Euclidian geometry as truth was an underpinning of mathematics until the nineteenth century, when mathematicians found that it was based on a flawed axiom. That axiom, the "Parallel Postulate," states that if you have a line and a point not on the line, only one line can be drawn through the point parallel to the first line. A trio of mathematicians — Lobachevsky, Bolyai, and Riemann — showed that there exists, in one case, more than one line; or in another, no such line. The truth of Euclidian geometry was either destroyed or transformed into three new truths, depending on your mood. Mathematicians preferred the three truths to no truth, and mathematical life went on.

Bertrand Russell continued the assault on mathematical truth in the twentieth century. Russell is famous for the nearly 2,000-page tome, *Principia Mathematica*, coauthored with Alfred North Whitehead, in which he attempted to reduce all mathematics to a form of logic. Russell used logic in the form of a paradox as his weapon against truth. Russell's Paradox, outlined in a letter to fellow mathematician Gottlob Frege, has an analogy in the statement by Epimenides, a Cretan, that "All Cretans are liars." Russell's mathematical statement of this paradox implied that there could be no truth in mathematics, since mathematical logic was flawed at a basic level.

This logical assault on mathematical truth continued in the work of Kurt Gödel, an esteemed associate of Albert Einstein. Gödel's Incompleteness Theorem, popularized by Douglas R. Hofstadter in his book, *Gödel, Escher, Bach: An Eternal Golden Braid* (Basic Books, New York, 1979), caused quite a stir at its publication. This theorem states that certain statements in mathematics exist in a shadow world in which they are neither true nor false; they are "undecidable." The consequence of this is that there is a fundamental uncertainty in mathematics.

Gödel's Incompleteness Theorem is to mathematics what the Heisenberg Uncertainty Principle is to physics. The Heisenberg Uncertainty Principle, published at about the same time as Gödel's Incompleteness Theorem, states that some things in the physical world cannot, in principle, be known. Many physicists, Einstein included, were not convinced that, in effect, some things Man was not meant to know. Today, the Heisenberg Uncertainty Principle is taken as fact, and it is even a useful tool. Likewise, mathematicians have chosen to live with incompleteness. Gregory J. Chaitin of IBM has stated that Gödel's Theorem "has had no lasting impact on the daily lives of mathematicians or on their working habits; no one loses sleep over it any more."

In 1950, Alan Turing, a founding father of computer science, proposed a test of machine intelligence that he called an "imitation game." This game is now called the "Turing Test," and the modern form has a person conversing with a computer program and guessing whether he or she is chatting with a machine or a real person. Turing thought it was necessary for the computer program to occasionally answer some questions wrong, lest its perfection prove it was not human. Mathematicians, of course, are human, and Andrew Hodges of Wadham College, Oxford, UK, remarks in the *Stanford Encyclopedia of Philosophy*, "Turing's post-war view was that mathematicians make mistakes, and so do not in fact see the truth infallibly. Once the possibility of mistakes is admitted, Gödel's theorem becomes irrelevant." Turing, apparently, rejected the idea of mathematical truth.

Computer programmers often repeat the slogan, "To err is human, but you need a computer to really screw things up." In light of the fallibility of computer code, it is interesting to see how some mathematicians have integrated computers into mathematics. The first notable example is the "proof" of the "Four-Color Conjecture." The Four-Color Conjecture dates back to 1852, when a mapmaker, Francis Guthrie, found that four colors seemed to suffice to make a map in which no region of any color abuts another of the same color. The mathematician Arthur Cayley published this as a conjecture in 1879, and in that same year Alfred Kempe published a proof that remained in force until 1890, when an error was discovered. In 1976, Kenneth Appel and Wolfgang Haken published a "proof" of the Four-Color Conjecture, pushing it into the domain of the Four-Color Theorem. Their proof caused considerable controversy because they had used a computer to analyze systematically every conceivable map and demonstrate that four colors worked. Appel and Haken's method can be realized only with a computer because the computational workload is too much for mere humans. Mathematicians, of course, are still searching for a "real proof" of the conjecture, but perhaps not as hard as they were before.

Now that the low-hanging fruits have been picked from the mathematics tree, the remaining conjectures are requiring very long proofs that are prone to error, both in their construction and in the necessary checking by other mathematicians. A mathematician might err in a proof, and other mathematicians might compound that error by not catching a mistake. Fermat's Last Theorem was, until recently, one of the great unproven conjectures. This conjecture was written by Fermat in the margin of a book around 1630, and Fermat claimed to have discovered a proof, but he did not have space to write it in the page margin. In their attempts to "rediscover" Fermat's proof, mathematicians have demonstrated their fallibility. J.J. O'Connor and E.F. Robertson of the University of St. Andrews (Scotland) have reported that a thousand false proofs of this conjecture were published between 1908 and 1912.

In 1993, Andrew Wiles of Princeton University announced another proof of Fermat's Last Theorem, and for a while it held the ring of truth. However, Wiles withdrew the proof at the end of the year when a gap in the logical flow of the proof could not be mended. Finally, in 1995, a corrected proof was produced with the aid of another mathematician, Richard Taylor. No errors have been discovered in this proof in the intervening years, and this gives it considerable, although not absolute, validity.

Mathematics is still prejudiced in favor of pencil-and-paper proof, but it is slowly making some concessions to computers. There is now a journal, *Experimental Mathematics*, published quarterly by A.K. Peters (Natick, Massachusetts) that mediates the fusion of computers and math. To quote from the journal's mission statement, "*Experimental Mathematics* was founded in the belief that theory and experiment feed on each other, and that the mathematical community stands to benefit from a more complete exposure to the experimental process." Does mathematical truth really exist, or will most of mathematics become a tentative consensus of a mathematical reality mediated by computers?

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